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Study and Analysis Capacity of MIMO Systems for AWGN Channel Model Scenarios

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Abstract
Future wireless communication systems can utilize the spatial properties of the wireless channel to enhance the spectral efficiency and therefore increases its channel capacity. This can be designed by deploying multiple antennas at both the transmitter side and receiver side. The basic measure of performance is the capacity of a channel; the maximum rate of communication for which arbitrarily small error probability can be achieved. The AWGN (additive white Gaussian noise) channel introduces the notion of capacity through a heuristic argument. The AWGN channel is then used as a basic building block to check the capacity of wireless fading channels in contrast to the AWGN channel. There is no single definition of capacity for fading channels that is applicable in all situations. Several notions of capacity are developed, and together they form a systematic study of performance limits of fading channels. The various capacity measures allow us to observe clearly the various types of resources available in fading channels: degrees of freedom, power and diversity. The MIMO systems capacity can be enhanced linearly with large the number of antennas. This paper elaborates the study of MIMO system capacity using the AWGN Channel Model, Channel Capacity, Channel Fast Fading, Spatial Autocorrelation and Power delay profile for various channel environments.
Keywords: MIMO, SISO, SIMO, MISO, AWGN, System Capacity and Waterfilling.

I. INTRODUCTION

MIMO (multiple-input multiple-output) is a multiple antenna technology for communication in wireless systems [2]. Multiple antennas are used at both the source (transmitter) and hence at the destination (receiver). The antennas at both the ends of the communications system are combined to reduce errors and maximize data rate. Wireless system performance depends mainly on the behaviour of the channel through which the signal passes. The transmitted signal encounters all kind of obstacles in the path. The multipath wireless environments give rise to constructive or destructive summation of the signal. This causes rapid fluctuation in signal nature causing its quality to be dropped down within this short span of time. This significant variation of wireless communication channel imposes strict limitation for reliable transmission. When the multipath component undergoes a phase shift of 2π over a distance as short as one wavelength, power fluctuation occurred by multipath over propagation for a very small time scale and therefore it is remarked to as small scale fading. Large scale fading results when these fluctuations occur over distances up to a few hundreds of wavelengths. From above discussion we can say that wireless propagation is generally governed by an immense variety of unpredictable factors which can be characterized as

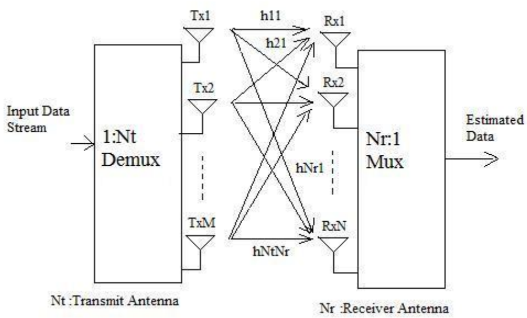

Fig. 1. Block diagram of MIMO system [2].

System Model

Let us assume narrow-band single user MIMO systems with N_T transmit and N_R receives antennas as depicted in Figure 1. The antennas are assumed to be omnidirectional, which implies that the antennas transmit and receive equally well in all directions. The linear link model between the transmit and receive antennas can be represented in the vector notation as

$$y = Hx + n \qquad (1)$$

where y is the $N_R \times 1$ received signal vector, x is the $N_T \times 1$ transmitted signal vector, n is the $N_R \times 1$ complex Gaussian noise vector with zero mean and equal variance, which isequal to σ^2,and H is the $N_R \times N_T$ normalized

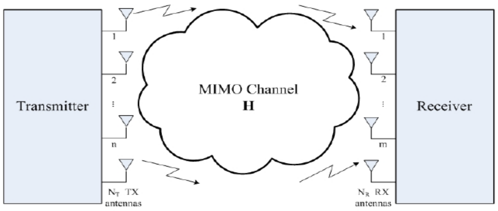

Fig. 2. Illustration of a simple MIMO system model [2].

channel matrix, which can be represented as

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_T} \\ h_{21} & h_{22} & \cdots & h_{2N_T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{N_{R_1}} & h_{N_{R_2}} & \cdots & h_{N_R N_T} \end{pmatrix} \quad (2)$$

Each element h_{mn} represents the complex gains between the nth transmit and mth receive antennas.

II. MIMO Capacity

In 1948, Claude Shannon started work on the channel capacity for additive white Gaussian noise (AWGN) channels [7]. Compared with the scalar AWGN channels, a MIMO system can offer drastic improvement to either communication quality (bit-error rate or BER) or transmission date rate (bits/sec) by making use of spatial diversity [3]. We also mentioned absolute capacity bounds, which compare SISO, single-input-multiple-output (SIMO) and multiple-input-single-output (MISO) capacities. As the feedback concept is a very essential part of communication system design, we further discuss a more special case which presumes a prior knowledge of the channel matrix at the transmitter. Before describing capacity, some assumptions need to be stated:

- In all these cases, we tend to concentrate on the single user form of capacity, so that the received signal is corrupted solely by additive white Gaussian noise.
- Capacity investigation depends on a “quasi-static” situation which implies that the channel is assumed fixed within a period of time (a burst), and also the burst is considered to be extended enough time length in which adequate bits are transmitted to formulate information theory be significant and meaningful [3, 5].
- The channels are considered to be memoryless channels which imply that each channel realization is independent from one another [4].

SISO System Capacity

The capacity for a memoryless SISO (Single-Input-Single-Output) system is given by [3]

$$C_{SISO} = \log_2(1+\rho\,|h|^2) \text{ bps/Hz} \quad (3)$$

where h is the normalized complex gain of the channel and ρ is the SNR at receiver antenna.

SIMO and MISO System Capacity

The system capacity with N_R RX antennas, the single-input-multiple-output (SIMO) is [3]

$$C_{SIMO} = \log_2(1+\rho\sum_{m=1}^{N_R}|h_m|^2) \text{ bps/Hz} \quad (4)$$

where h_m is the gain for, m^{th} RX antenna. Moreover, if N_T TX antennas are utilized, multiple-input-single-input (MISO) properties can be achieved. The capacity is given as [3]

$$C_{MISO} = \log_2(1+\frac{\rho}{N_T}\sum_{n=1}^{N_T}|h_n|^2) \text{ bps/Hz} \quad (5)$$

where h_n is the gain for nth TX antenna. To ensure the transmitter power restriction, SNR is normalized by N_T.

Figure 3 illustrates the capacity comparison of SISO, SIMO and MISO system versus SNR. From the figure, we can observe that the SIMO and MISO channels achieved much better capacity compared with the SISO channel by exploiting more antennas. However, the SIMO and MISO channels can only offer a logarithmic increase in capacity with the number of antennas [1]. It is clear that CMISO<CSIMO when the channel information is not available at the transmitter [1].

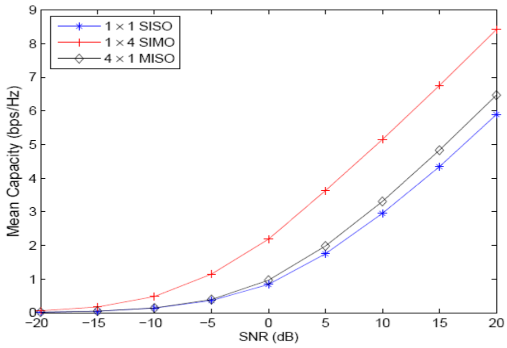

Fig. 3. Mean capacity comparison of SISO, SIMO and MISO systems as a function of SNR.

MIMO System Capacity with Equal Power

For N_T Transmit and N_R Receive antennas, the equal power capacity equation is [4, 5]

$$C_{EP} = \log_2\left[\det\left(I + \frac{\rho}{N_T}HH^*\right)\right] \text{ bps/Hz} \quad (6)$$

where det(.) depicts the determinant of a matrix, I is an $N_R \times N_T$ identity matrix, ρ is the average received Signal to Noise Ratio (SNR), and H* is the complex conjugate transpose of H.

MIMO System Capacity with Waterfilling

If the channel knowledge is not available at the transmitter side; the individual sub channels cannot be assessed. So that equal power allocation is logical under this situation.

Using the principle of Waterfilling model the channel capacity can be given as:

$$C_{WF} = \sum_{i=1}^{m} \log_2 \left(\mu\lambda_i\right)^{*} \text{ Bps/Hz} \qquad (7)$$

where μ is chosen from the waterfilling algorithm, which is

$$\rho = \sum_{i=1}^{m} \left(\mu - \lambda_i^{-1}\right)^{*} (8)$$

where (.)* denotes taking only those terms which are positive and $\lambda_1, \lambda_2, \ldots, \lambda_m$ are the eigenvalues of W with $m = \min(N_T, N_R)$.

Compared with the equal power scheme, waterfilling has a significant advantage especially at low SNR. However, this advantage decreases as SNR is increased [6]. Figure 4 illustrates the waterfilling concept.

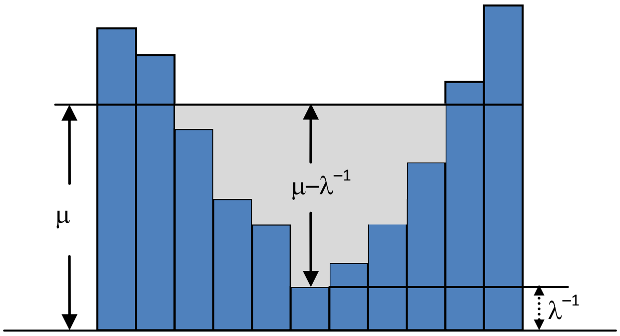

Fig. 4. Waterfilling Concept [13]

III. MIMO Capacity and gain of Optimal power allocation for AWGN Channel

GAUSSIAN CHANNELS

Let us consider m independent Gaussian channels in parallel with a common power constraint. The main motive behind is to achieve an equal distribution of the total power among the channels so as to optimize the capacity. This channel models a non-white additive Gaussian noise channel where every individual parallel component represents a different frequency.

Let us consider a set of Gaussian channels in parallel as represented in Figure 5. The output of each channel is the summation of the input and Gaussian noise. For channel j,

$Y_j = X_j + Z_j$, $\quad j = 1, 2, \ldots, m,$ (9)

with

$Z_j \sim N(0, N_j)$ (10)

and the noise is assumed to be independent from channel to channel. We assume that there is a common power constraint on the total power used i.e.

$$E\sum_{j=1}^{m} X_j^2 \le P \qquad (11)$$

In order to maximize the total capacity; it is recommended to distribute the power among the various parallel channels in a given model.

Thus the channel's information capacity can be given as:

$$C = \max_{f(x_1, x_2, \ldots x_m);\, \sum \mathrm{E}X_i^2 \le P} I(X_1, X_2, \ldots X_m; Y_1, Y_2, \ldots Y_m) (12)$$

PARALLEL GAUSSIAN CHANNELS

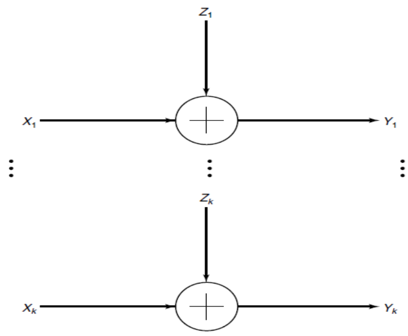

Fig. 5. Parallel Gaussian channels.

Here we compute the distribution that achieves the information capacity for the above channel. The actual scenario is that the information capacity is the supermom of achievable rates can be proved by techniques same as applied in the proof of the capacity theorem for single Gaussian channels and can be avoided.

Since $Z_1, Z_2, \ldots, Z_m$ are independent,

$I(X_1, X_2, \ldots, X_m; Y_1, Y_2, \ldots, Y_m)$

$= h(Y_1, Y_2, \ldots, Y_m) - h(Y_1, Y_2, \ldots, Y_m | X_1, X_2, \ldots, X_m)$

$= h(Y_1, Y_2, \ldots, Y_m) - h(Z_1, Z_2, \ldots, Z_m | X_1, X_2, \ldots, X_m)$

$= h(Y_1, Y_2, \ldots, Y_m) - h(Z_1, Z_2, \ldots, Z_m)$ $= h(Y1, Y2, \ldots, Ym) - \sum h(Z_i)$

$$\le \sum_i h(Y_i) - h(Z_i)$$

$$\le \sum_i \frac{1}{2}\log\left(1+\frac{P_i}{N_i}\right), \qquad (13)$$

where $P_i = EX_i^2$, and $\Sigma P_i = P$. Equality is achieved by

$$(X_1, X_2, \ldots, X_m) \sim N\left(0, \begin{bmatrix} P_1 & 0 & \cdots & 0 \\ 0 & P_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_m \end{bmatrix}\right) (14)$$

Thus we observed that the problem is reduced to finding the power allotment that optimizes the channel capacity subject to the constraint that $\sum P_i = P$. This problem is a standard optimization problem which can be solved using Lagrange multipliers.

This solution is represented graphically in Figure 6. The vertical levels depict the noise levels in the various channels. Initially when the signal power rises from zero, we will first allot the power to the channels with the lowest noise. If the available power is increased further, we can allot some of the power to noisier channels. The process by which the power is distributed among the various bins is much similar to the way in which water distributes itself in a container; hence this process is sometimes known as water-filling technique.

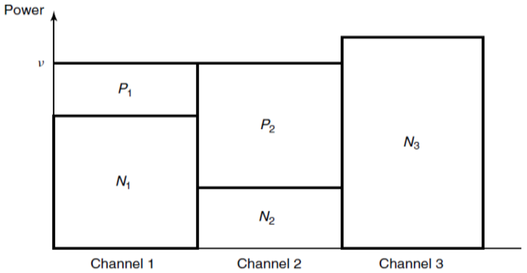

Fig. 6. Illustration of Water-filling for parallel channels.

IV. Simulation and Results Analysis

In this study, using water filling channel model we made a comparison among the capacity of parallel Gaussian channels for equal power allocation and optimal power allocation. Also for SISO, SIMO, MISO and MIMO; we made an analytical comparisons for the channel parameters such as Mean capacity, Complementary distribution function CDF and Outage probability using Water-Filling power allocation. The simulation is carried out using MATLAB software.

We assume the Simulation Parameters as follows:

SNR_dB=-10:30;
SNR=10.^(SNR_dB./10);
ch_realizations=10000; *(Monte Carlo sim. of 10,000 channel realizations)*
c_outage=4;
Epsilon=1e-6.

Case: Parallel Gaussian Channels

Let the received signal be:

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} (15)$$

where

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} \sim N\left(0, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}\right) (16)$$

Capacity when power is equally distributed between users:

$$C = \sum_{i=1}^{4} \frac{1}{2} \log\left(1 + \frac{P_i}{N_i}\right) (17)$$

$$= \frac{1}{2}\left[\log_2(1+5) + \log_2(1+5/7) + \log_2(1+1) + \log_2(1+5/3)\right]$$

$= 2.8888 \cong 2.9$

By running the water filling algorithm, we get:

For $N_1 = 1 \rightarrow P_1 = 8$

For $N_2 = 7 \rightarrow P_2 = 2$

For $N_3 = 5 \rightarrow P_3 = 4$

For $N_4 = 3 \rightarrow P_4 = 6$

$\Rightarrow C = 3 > 2.88$

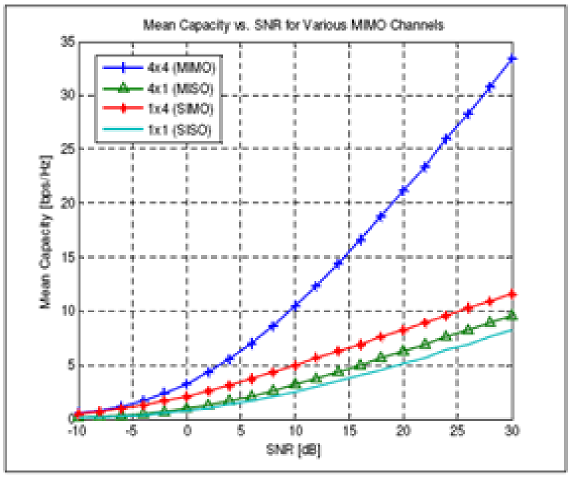

Figure 7: Mean Capacity v/s SNR

Figure 7 depicts the detailed analysis of mean capacity over a wide range of SNR for different receiver antenna configuration. We studied various MIMO channels configuration over these SNR and found out that mean capacity of a 4 × 4 MIMO channel increases exponentially against SNR and are far better with respect to any other configuration of a MIMO antenna. Thus 4 × 4 MIMO channel surely assists in sending large data packets without any packet loss as the capacity of the channel in this mode is highest with respect to other configuration.

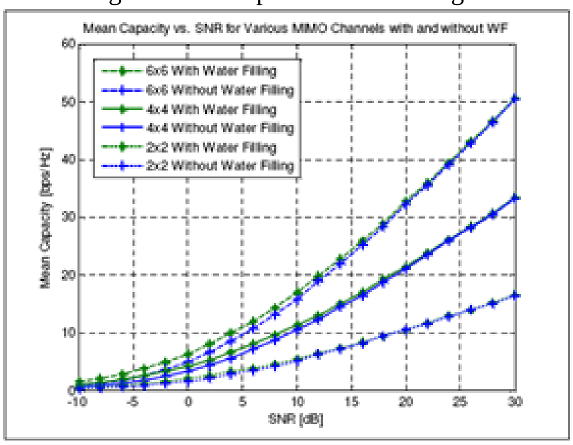

Figure 8: Mean Capacity v/s SNR with and without WF

In the above figure 8 it is clearly depictable that mean channel capacity of 6 × 6 MIMO channel for lower values of SNR in waterfilling model is high with respect to 6 × 6 MIMO channel without waterfilling model. This result clearly signifies that the power division in higher mode of MIMO channel is properly utilized than lower modes and thus proves that N × N MIMO channels with larger values of N possess high mean channel capacity. The channel capacity enhances significantly with SNR.

V. CONCLUSION

Finally we can conclude that, the capacity increases linearly with the number of antennas for the case of AWGN channel and the performance of water filling is far better than the equal power allocation scheme for low SNR value and this gap can be minimized; if SNR increases. In a nutshell, after reviewing all results it can be clearly stated that the power division concept as per the WF model for a MIMO channel significantly increases system efficiency than a normal case and hence it could be a novel approach to look at the system for better results in a MIMO environment.

REFERENCES

[1] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communication*. Cambridge ; New York : Cambridge University Press, 05.

[2] David TseandPramodViswanath, "*Fundamentals of Wireless Communication*"2nd ed. Cambridge Press, 2005.

[3] D. Gesbert, M. Shafi, D. shanShiu, P. J. Smith, and A. Naguib, "*From theory to practice: an overview of MIMO space-time coded wireless systems,"* IEEE J. Select. Areas Commun., vol. 21, no. 3, pp. 281–302, Apr. 2003.

[4] I. E. Telatar, "*Capacity of multi-antenna Gaussian channels*," Office for Official Publications of the European Communities, AT&T Bell Labs., Tech. Rep. BL0112170-950615-07TM, 1995.

[5] G. J. Foschini and M. J. Gans, "*On limits of wireless communications in a fading environment when using multiple antennas*," Wireless Pers. Comm., vol. 6, no. 3, pp.311–335, Mar. 1998.

[6] P. J. Smith and M. Shafi, "*Waterfilling methods for MIMO systems*," in 3rd Australian Communications Theory Workshop (Aus CTW), Canberra, Australia, Feb. 4-5, 2002,pp. 33–37.

[7] C. Shannon, "*A mathematical theory of communication," Bell Labs Technical Journal*, vol. 27, pp. 379–423,623–656, July and October 1948.